Journal of Computational Physics **157,** 796–800 (2000)
doi:10.1006/jcph.1999.6392, available online at http://www.idealibrary.com on IDEAL®

NOTE

A Simple Method for Computing Far-Field Sound in Aeroacoustic Computations

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Received October 7, 1999

1. INTRODUCTION

When computing aerodynamic noise from moderate Mach number flows, compressible flow equations are often solved numerically in a finite region of space containing the noise sources and the near acoustic field [1]. However, whether this is done by direct numerical simulation or with some modeling approximations (e.g., large-eddy simulation or unsteady Reynolds averaging), such simulations typically involve a range of length and time scales and are therefore expensive. To cope with this, the computational domain is often truncated in an acoustic region that is not too far beyond the unsteady flow. The sound at greater distances is then computed by solving relatively simple acoustic equations.

Several numerical methods for extending the solution in this fashion are reviewed by Shih *et al*. [2]. One approach is to use wave equation solutions formulated as surface integrals, so-called Kirchhoff or Ffowcs Williams–Hawkings methods [3–5]. The integral evaluation operations scale as $O(N^2)$ per "measurement." So, if the acoustic field is needed at a series of N times in an N^3 volume, the total scaling is $O(N^6)$. Providing this data is at times impractical due to computer memory limitations; and, though binning techniques are available, application is complicated by the fact that data are required at retarded times.

When extensive sound field data are needed it is at times better to solve a wave equation (or similar set of acoustic equations) into the far field using direct methods. After an initial transient, this approach scales like $O(N^4)$ (three spatial dimensions plus time) for $O(N^4)$ measurements.[1] The implementation of the direct solver is simpler because the data are now only needed sequentially. Existing direct methods are formulated either as an

[1] Note that there may be a large coefficient in this case if the required measurements are considerably separated in time.

0021-9991/00 $35.00

inhomogeneous wave equation (acoustic analogy), where a sound source is explicitly computed from the flow data, or as an unsteady boundary value problem, where data are provided from the compressible flow computation on a common boundary [2]. Boundary condition formulations tend to be less general because the location and geometry of the common boundary between the flow and acoustic regions are problem dependent.

The method proposed in this note is classified as a direct method and so is not hampered by the data management issues discussed above. It is novel because it requires neither an acoustic source to be computed nor a matching between the flow and acoustic solutions. Its principal strength is its simplicity, but its computational cost is reasonable and, as discussed above, can be advantageous when extensive acoustic data are required.

2. MATHEMATICAL FORMULATION

Consider a region of flow, Ω_s, that contains all the pertinent acoustic sources. Ω_s is a subregion of the computational domain, Ω_c, where the compressible flow equations are solved. The objective is to calculate the sound at a distance from Ω_c.

A method for doing this might utilize the fact that solutions of the flow equations also satisfy Lighthill's equation [6],

$$\frac{\partial^2 \rho}{\partial t^2} - a_\infty^2 \frac{\partial^2 \rho}{\partial x_j \partial x_j} = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}, \tag{1}$$

where $T_{ij} = \rho u_i u_j + (p - a_\infty^2 \rho)\delta_{ij} - \tau_{ij}$, and a_∞ is the sound speed, ρ is the density, p is the pressure, u_i is the velocity, and τ_{ij} is the viscous stress tensor. Given T_{ij} at all points in the flow, (1) can be solved for the sound field.

An alternative method proposed here, that does not require T_{ij}, is to solve linearized Euler equations with an additional term,

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \frac{\partial u_j}{\partial x_j} &= -\sigma(\rho - \rho_{NS}) \\ \frac{\partial u_i}{\partial t} + a_\infty^2 \frac{\partial \rho}{\partial x_i} &= 0, \end{aligned} \tag{2}$$

or, equivalently, the wave equation

$$\frac{\partial^2 \rho}{\partial t^2} - a_\infty^2 \frac{\partial^2 \rho}{\partial x_j \partial x_j} = -\sigma\left(\frac{\partial \rho}{\partial t} - \frac{\partial \rho_{NS}}{\partial t}\right). \tag{3}$$

In (2) and (3), ρ_{NS} is the density from the Navier–Stokes solution in Ω_c and σ is a function of space that is large in Ω_s but has compact support in Ω_c. The purpose of the new term is to drive the acoustic density toward the Navier–Stokes value. Note that the density (or $\partial_t \rho_{NS}$) from the Navier–Stokes solution is the only input required from the flow solver.

To demonstrate how the method works and determine requirements on σ a one-dimensional advection equation is considered that has obvious similarities to (2) and (3),

$$\frac{\partial \rho}{\partial t} + a_\infty \frac{\partial \rho}{\partial x} = -\sigma(x)(\rho - \rho_{NS}), \qquad x > x_s,\ t > 0,\ \rho(x_s, t) = \rho_o(t). \tag{4}$$

The point $x_s \in \partial\Omega_s$ represents the outer extent of the unsteady aerodynamic source flow. The function σ is such that $\sigma(x) = 0$ for $x > x_c$, where $x_c \in \partial\Omega_c$ corresponds to the outer edge of the computation domain. Our objective is to demonstrate that $\rho(x,t) \to \rho_{NS}(x,t)$ for $x > x_c$ regardless of $\rho_o(t)$, where ρ_o represents aerodynamic density fluctuations.

The solution of (4) for $x > x_c$ is

$$\rho(x,t) = \rho_{NS}(x,t) + [\rho_o(x - a_\infty t) - \rho_{NS}(x,t)] \exp\left[-\frac{1}{a_\infty}\int_{x_s}^{x_c} \sigma(\xi)\, d\xi\right]. \tag{5}$$

Clearly, $\rho(x,t) \to \rho_{NS}(x,t)$ as the exponential factor approaches zero, and σ, x_s, and x_c must be chosen to give the desired degree of accuracy. Large values for σ (or long distances between $\partial\Omega_c$ and $\partial\Omega_s$) will remove the influence of ρ_o more effectively. The accuracy of the method will depend upon the relative amplitude of near-field and acoustic field density fluctuations. These may be used to estimate the requirements on σ by (5). A more precise measure would be to compute $\rho - \rho_{NS}$ on $\partial\Omega_c$ and determine if the desired accuracy has been achieved. Equation (5) then provides guidance for improving the accuracy.

Additional insight can be gained by solving (3). To do this a Fourier transform is defined

$$\hat{\rho}(\mathbf{k},\omega) = \frac{1}{16\pi^4}\int_{-\infty}^{\infty} \rho(\mathbf{x},t) e^{i\mathbf{k}\cdot\mathbf{x}} e^{i\omega t}\, d\mathbf{x}\, dt. \tag{6}$$

The transformed Navier–Stokes density, $\hat{\rho}_{NS}$, is similarly defined. Applying (6) to (3) yields

$$\left(-\omega^2 + i\omega\sigma + a_\infty^2 k^2\right)\hat{\rho} = i\omega\sigma\,\hat{\rho}_{NS}, \tag{7}$$

where $k = |\mathbf{k}|$. Inverse transforming gives

$$\rho(\mathbf{x},t) = \int_{-\infty}^{\infty} \frac{\hat{\rho}_{NS}}{1 - (i/\sigma)\left(a_\infty^2 k^2/\omega - \omega\right)} e^{-i\mathbf{k}\cdot\mathbf{x}} e^{-i\omega t}\, d\mathbf{k}\, d\omega. \tag{8}$$

Expanding the denominator for large σ gives

$$\rho(\mathbf{x},t) = \rho_{NS}(\mathbf{x},t) + \int_{-\infty}^{\infty} \hat{\rho}_{NS}\frac{i}{\sigma}\left(\frac{a_\infty^2 k^2}{\omega} - \omega\right) e^{-i\mathbf{k}\cdot\mathbf{x}} e^{-i\omega t}\, d\mathbf{k}\, d\omega + O(\sigma^{-2}, \omega^2). \tag{9}$$

So to first order in σ^{-1}, $\rho = \rho_{NS}$ within Ω_s. Beyond Ω_s there are no sources and the flow is acoustic so the solution would be the same in the $\sigma \to \infty$ limit if the compressible flow solver were extended.

By time differentiating (9), it can be shown that the right hand side of (3) is an approximate representation of the Lighthill source,

$$\begin{aligned}\sigma\left(\frac{\partial\rho}{\partial t} - \frac{\partial\rho_{NS}}{\partial t}\right) &= i\frac{\partial}{\partial t}\int_{-\infty}^{\infty} \hat{\rho}_{NS}\left(\frac{a_\infty^2 k^2}{\omega} - \omega\right) e^{-i\mathbf{k}\cdot\mathbf{x}} e^{-i\omega t}\, d\mathbf{k}\, d\omega + O(\sigma^{-1}, \omega^3)\\ &= \int_{-\infty}^{\infty} \hat{\rho}_{NS}\left(a_\infty^2 k^2 - \omega^2\right) e^{-i\mathbf{k}\cdot\mathbf{x}} e^{-i\omega t}\, d\mathbf{k}\, d\omega + O(\sigma^{-1}, \omega^3)\\ &= \frac{\partial^2\rho_{NS}}{\partial t^2} - a_\infty^2\Delta\rho_{NS} + O(\sigma^{-1}, \omega^3) = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} + O(\sigma^{-1}, \omega^3).\end{aligned} \tag{10}$$

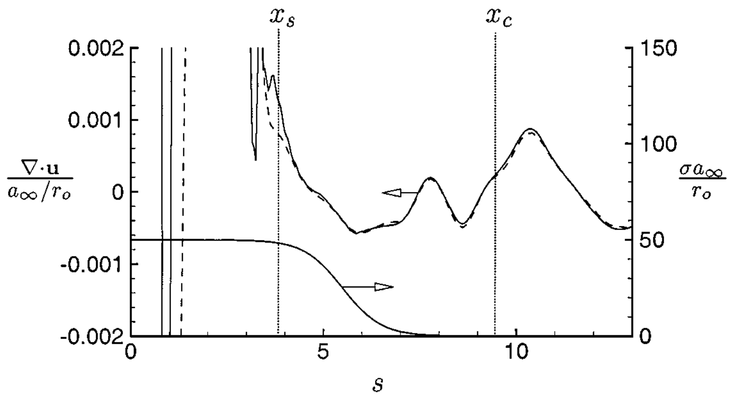

FIG. 1. Dilatation ($= -\partial_t \rho$) from the flow solver and from the solution of (3) on a ray inclined at 60° from the end of the potential core: —, direct numerical simulation; - - -, proposed method. s is the distance along the ray. The σ-factor is shown on the right axis, and the effective $x_s \in \partial\Omega_s$ and $x_c \in \partial\Omega_c$ are labeled.

This result shows that in some sense the proposed method is more similar to an acoustic analogy than a Kirchhoff or matched boundary method. It is therefore not so sensitive to open integral surface or matched boundary necessary in some applications.

3. A RESULT

The results of a successful application of the proposed technique to compute the noise from a subsonic round jet will be reported elsewhere [7]. To illustrate the method in this application, the instantaneous dilatation ($= -\partial_t \rho$) on a ray extending from the jet is shown here in Fig. 1. The directly computed solution and the solution of (3) agree well outside the turbulent flow. They deviate from each other within the source where the time rate of change of density is not necessarily equal to the negative dilatation. This was modeled as ρ_o in (4). To converge overall sound pressure level statistics on an arc 60 jet radii from the nozzle, the jet simulation required approximately 5,000 node hours on a 175 MHz IBM SP, while the far-field sound required only 25 node hours on a cluster of 500 MHz Alpha-based PCs. The expense of the far-field sound computation was, therefore, essentially negligible.

4. DISCUSSION

As discussed in the Introduction, the proposed method will be most useful when a large amount of sound field data are needed at a not too great of distance from the source. For long-range propagation the expense increases considerably as will numerical errors that accumulate with distance. For these reasons semi-analytical methods whose cost and accuracy are not so constrained by the distance between the source and "measurement" point might in some cases be preferable despite the data management issues discussed in Section 1. It should also be noted that the proposed method is only useful when it is feasible to solve compressible equations for the sound. For very low Mach of number flow, where the length and time scales are such that compressible flow computations are impractical, incompressible solutions in conjunction with acoustic analogies are probably advantageous.

While the jet application discussed above was successful, aeroacoustic problems are varied and the issue of generality is difficult to address. For example, it is easy to anticipate situations where a mean flow might exist in the acoustic region. Examining the proper modification of (2) for this case will be important and applicability to this case is potentially a strength of the method. It might also be possible to include nonlinear effects in the "acoustic" equations. Such extensions and the evaluation of the method for specific aeroacoustic problems will be the subject of future work.

5. CONCLUSIONS

A simple new method has been proposed for computing far-field sound in conjunction with near-field aeroacoustic computations. The only input necessary from the flow computation is the density; no acoustic sources need to be computed. Furthermore, the density data are only required at a series of discrete times, rather than a range of retarded times as required by many methods. The proposed method has been successfully demonstrated in the context of a jet simulation.

ACKNOWLEDGMENT

Support from NASA is gratefully acknowledged.

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